

TEBRANISHLAR VA TO'LQINLAR

Masalalar

51-masala. Moddiy nuqtaning tebranishi $x = 0,25 \sin\left(\pi t + \frac{\pi}{2}\right)m$ qonun bo'yicha bajariladi. Tebranish amplitudasi A , davri T , siklik chastotasi ω , boshlang'ich fazasi φ_0 maksimal tezligi v_{\max} va maksimal tezlanishi a_{\max} topilsin.

Berilgan: $x = 0,25 \sin\left(\pi t + \frac{\pi}{2}\right)m$

$A \sim ? \quad T \sim ? \quad \omega \sim ? \quad \varphi_0 \sim ? \quad v_{\max} \sim ? \quad a_{\max} \sim ?$

Echish. Topilishi kerak bo'lgan kattaliklarni aniqlash uchun tebranishning tenglamasini garmonik tebranishning umumiy ko'rinishidagi tenglamasi bilan solishtiramiz:

$$x = 0,25 \sin\left(\pi t + \frac{\pi}{2}\right)m$$
$$x = A \sin\left(\frac{2\pi}{T}t + \varphi_0\right)m$$

Bu ikki tenglama taqqoslanishidan quyidagi kelib chiqadi: tebranishning amplitudasi $A=0,25m$; davri $\frac{2\pi}{T}t = \pi t$, bundan $T=2s$; siklik chastotasi

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = 3,14 \text{ rad/s}; \text{ boshlang'ich fazasi } \varphi_0 = \frac{\pi}{2}:$$

Tebranishning tezligi v va tezlanishi a mos ravishda siljish funksiyasining birinchi va ikkinchi tartibli hosilasidan iborat bo'lgani uchun:

$$v = \frac{dx}{dt} = 0,25\pi \cos\left(\pi t + \frac{\pi}{2}\right) \text{ bo'lib, } v_{\max} = 0,25\pi m/s = 0,785 m/s. \text{ SHunday qilib,}$$

$$v_{\max} = 0,785 m/s, \quad a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = -0,25\pi^2 \sin\left(\pi t + \frac{\pi}{2}\right) \text{ bo'lib, } a = -0,25\pi^2 m/s^2$$

$$a = -0,25 \cdot 3,14^2 m/s^2 = -2,46 m/s^2. \text{ SHunday qilib } a = -2,46 m/s^2.$$

52-masala. $m=5$ g massali moddiy nuqta $\nu=0,5Hz$ chastota bilan garmonik tebranadi. Tebranish amplitudasi $A=3sm$. 1) nuqtaning siljishi $x=1,5$ sm bo'lgan vaqtdagi tezligi v ; 2) nuqtaga ta'sir etuvchi maksimal kuch F_{\max} ; 3) tebranayotgan nuqtaning to'liq energiyasi W aniqlansin.

Berilgan: $\nu = 0,5Hz, m = 5g = 5 \cdot 10^{-3} kg, A = 3sm = 3 \cdot 10^{-2} m$

$$x = 1,5sm = 1,5 \cdot 10^{-2} m$$

$v \sim ?, F_{\max} \sim ?, W \sim ?$

Echish. 1) garmonik tebranish tenglamasi quyidagi ko'rinishga ega

$$x = A \cos(\omega t + \varphi) \quad (1)$$

Tezlik formulasini esa siljishidan vaqt bo'yicha birinchi tartibli hosila olib topamiz:

$$v = \frac{dx}{dt} = -A\omega \sin(\omega t + \varphi) \quad (2)$$

Tezlikni siljish orqali ifodalash uchun (1) va (2) tenglamalardan vaqtni yo'qotish kerak. Buning uchun har ikkala tenglamani kvadratga ko'tarib, birinчисini A ga, ikkinчисini $A^2\omega^2$ ga bo'lamiz va ularni qo'shamiz:

$$\frac{x^2}{A^2} + \frac{v^2}{A^2\omega^2} = 1 \quad \text{yoki} \quad \frac{x^2}{A^2} + \frac{v^2}{4\pi^2\nu^2 A^2} = 1$$

Oxirgi tenglamani v ga nisbatan yechib, quyidagini topamiz:

$$v = \pm 2\pi\nu\sqrt{A^2 - x^2}$$

SHu formula bo'yicha hisoblashni bajarsak

$$v = \pm 8,2 \text{ sm/s}$$

2) nuqtaga ta'sir etuvchi kuchni Nyutonning ikkinchi qonuniga binoan topamiz

$$F = ma \quad (3)$$

Bunda a – nuqtaning tezligidan vaqt bo'yicha hosila olib topiladigan tezlanishi

$$a = \frac{dv}{dt} = -A\omega \cos(\omega t + \varphi) \quad \text{yoki} \quad a = -4\pi^2\nu^2 A \cos(\omega t + \varphi)$$

tezlanishning ifodasini (3) formulaga qo'ysak:

$$F = -4\pi^2\nu^2 mA \cos(\omega t + \varphi)$$

Bundan kuchni maksimal qiymati.

$$F_{\max} = 4\pi^2\nu^2 mA$$

Bu tenglamaga π, ν, m va A kattaliklarning qiymatlarini qo'ysak,

$$F_{\max} = 1,49 \text{ mN}$$

3) tebranayotgan nuqtaning to'liq energiyasi istalgan vaqt oralig'i uchun kinetik va potensial energiyalarning yig'indisiga tengdir.

To'liq energiyani hisoblashning eng sodda yo'li uni kinetik energiya potensial energiya maksimal qiymatga erishganda hisoblashdir. Bu vaqtda potensial energiya nolga teng bo'ladi (yoki kinetik energiya). SHuning uchun ham tebranayotgan nuqtaning to'liq energiyasi W maksimal kinetik energiya $W_{k\max}$ ga teng bo'ladi:

$$W = W_{k\max} = \frac{1}{2} m v_{\max}^2 \quad (4)$$

Maksimal tezlik (2) formulaga asosan $\sin(\omega t + \varphi) = -1$ qo'yib

$$v_{\max} = 2\pi\nu A$$

Tezlikning ifodasini (4) formulaga qo'ysak

$$W = 2\pi^2 m \nu^2 A^2$$

Kattaliklarning qiymatlarini bu formulaga qo'yib hisoblaymiz:

$$W = 2 \cdot (3,14)^2 \cdot 5 \cdot 10^{-3} \cdot (0,5)^2 \cdot (3 \cdot 10^{-2})^2 \text{ J} = 22,1 \cdot 10^{-6} \text{ J} = 22,1 \text{ mkJ}$$

53-masala. $x_1 = A_1 \cos \omega(t + \tau_1); x_2 = A_2 \cos \omega(t + \tau_2)$ tenglamalar bilan ifodalanadigan, bir xil yo`nalishli ikkita tebranish qo`shiladi. Bunda $A_1=1 \text{ sm}, A_2=2 \text{ sm}$

$$\tau_1 = \frac{1}{6} \text{ s}, \tau_2 = \frac{1}{2} \text{ s}, \omega = \pi \text{ s}^{-1}$$

1) qo`shiluvchi tebranishlarning boshlang`ich fazalari φ_1 va φ_2 lar aniqlansin; 2) natijaviy tebranishning amplitudasi A va boshlang`ich fazasi φ topilsin. Natijaviy tebranishning tenglamasi yozilsin.

Berilgan: $A_1=1 \text{ sm} = 1 \cdot 10^{-2} \text{ m}, A_2=2 \text{ sm} = 2 \cdot 10^{-2} \text{ m},$

$$\tau_1 = \frac{1}{6} \text{ s}, \tau_2 = \frac{1}{2} \text{ s}, \omega = \pi \text{ s}^{-1}$$

$$\varphi_1 \sim ?, \varphi_2 \sim ? \varphi \sim ? A \sim ?$$

Echish. 1. Garmonik tebranishning tenglamasi

$$x = A \cos(\omega t + \varphi) \quad (1)$$

ko`rinishga ega. Masala shartida berilgan tenglamalarni (1) ko`rinishga keltiramiz

$$x_1 = A_1 \cos \omega(t + \tau_1); x_2 = A_2 \cos \omega(t + \tau_2) \quad (2)$$

(2) ifodadan (1) tenglik bilan solishtirishdan birinchi va ikkinchi tebranishlarning boshlang`ich fazalarini topamiz:

$$\varphi_1 = \omega \tau_1 = \frac{\pi}{6} \text{ rad} \text{ va } \varphi_2 = \omega \tau_2 = \frac{\pi}{2} \text{ rad}$$

2) natijaviy tebranishning amplitudasi A ni aniqlash uchun kosinuslar teoremasidan foydalanamiz (14,4-§)

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \Delta \varphi} \quad (3)$$

bu yerda $\Delta \varphi$ - qo`shiluvchi tebranishlarning fazalar farqi $\Delta \varphi = \varphi_2 - \varphi_1$ bo`lganligidan, φ_2 va φ_1 larning topilgan qiymatlarini o`rniga qo`ysak,

$$\Delta \varphi = \frac{\pi}{3} \text{ rad}$$

A_1, A_2 va φ_1, φ_2 larning qiymatlarini (3) formulaga qo`yib hisoblasak

$$A = 2,65 \text{ sm}$$

Natijaviy tebranishning boshlang`ich fazasi tangensini (14,4-§) dagi 14,7 rasmdan aniqlaymiz

$$\text{tg } \varphi = \frac{A_1 \sin \varphi_1 + A_2 \sin \varphi_2}{A_1 \cos \varphi_1 + A_2 \cos \varphi_2} \text{ bundan boshlang`ich faza}$$

$$\varphi = \text{arctg} \frac{A_1 \sin \varphi_1 + A_2 \sin \varphi_2}{A_1 \cos \varphi_1 + A_2 \cos \varphi_2}$$

A_1, A_2, φ_1 va φ_2 larning qiymatlarini qo`yamiz va hisoblaymiz:

$$\varphi = \text{arctg} \left(\frac{5}{\sqrt{3}} \right) = 70,9 = 0,394 \pi \text{ rad}$$

54-masala. Moddiy nuqta bir paytning o`zida tenglamalari

$$x = A_1 \cos \omega t \quad (1)$$

$$y = A_2 \cos \frac{\omega}{2} t \quad (2)$$

ko`rinishda bo`lgan ikkita o`zaro tik garmonik tebranishda ishtirok etadi. Bunda $A_1=1\text{sm}$, $A_2=2\text{sm}$, $\omega = \pi\text{s}^{-1}$. Nuqta traektoriyasining tenglamasi topilsin.

Berilgan: $A_1 = 1\text{sm} = 1 \cdot 10^{-2}\text{m}$, $A_2 = 2\text{sm} = 2 \cdot 10^{-2}\text{m}$, $\omega = \pi\text{s}^{-1}$
Traektoriya tenglamasi ~?

Echish. Nuqta traektoriyasining tenglamasini topish uchun, berilgan (1) va (2) tenglamalaridan t vaqtini yo`qotamiz. Bu maqsadda $\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1+\cos\alpha}{2}}$ formulasidan foydalanamiz. U holda $\alpha = \omega t$, shuning uchun

$$y = A_2 \cos \frac{\omega}{2} t = A_2 \sqrt{\frac{1+\cos\omega t}{2}}$$

(1) formulaga binoan $\cos\omega t = \frac{x}{A_1}$ ekanligidan, traektoriya tenglamasi

$$y = A_2 \sqrt{\frac{1+\frac{x}{A_1}}{2}} \quad (3)$$

hosil bo`lgan ifoda o`qi OX o`qi bilan mos keluvchi parabola tenglamasidir.

Mustaqil yechish uchun masalalar

157. Amplitudasi $A=0,1\text{m}$, davri $T=4\text{ s}$ va boshlang`ich fazasi nolga teng bo`lgan garmonik tebranma harakat tenglamasini yozing. ($x=0,1\sin 0,5\pi t$ m).

158. Garmonik tebranish amplitudasi $A=5\text{sm}$, davri $T=4\text{ s}$ ga teng. Tebranayotgan nuqtaning maksimal tezligi va uning maksimal tezlanishini toping. ($v_{\max}=7,85 \cdot 10^{-2}\text{m/s}$; $a_{\max}=12,3 \cdot 10^{-2}\text{m/s}^2$).

159. Garmonik tebranishning boshlang`ich fazasi nolga teng. Nuqta muvozanat vaziyatidan $x_1=2,4\text{ sm}$ siljiganda nuqtaning tezligi $v_1=3\text{sm/s}$ bo`ladi, $x_2=2,8\text{ sm}$ siljiganda esa nuqtaning tezligi $v_2=2\text{sm/s}$ bo`ladi. SHu tebranishning amplitudasi va davri topilsin. ($A=3,1 \cdot 10^{-2}\text{m}$; $T=4,1\text{s}$).

160. Garmonik tebranma harakat qilayotgan jismning to`la energiyasi $W=3 \cdot 10^{-5}\text{J}$, jismga ta`sir etuvchi maksimal kuch $F=1,5 \cdot 10^{-3}\text{N}$ ga teng. Tebranish davri $T=2\text{ s}$ va boshlang`ich faza $\varphi=60^\circ$ bo`lsa, bu jismning harakat tenglamasini yozing. $\left[x = 0,04 \sin\left(\pi t + \frac{\pi}{3}\right) \text{m} \right]$.

161. Prujinaga $R=98,1\text{N}$ yuk osilgan. Prujina $F=9,8\text{ N}$ kuch ta`sirida $\Delta x=1,5\text{sm}$ cho`zilishi ma`lum bo`lsa, yukning vertikal tebranish davrini aniqlang. ($T=0,78\text{s}$).

162. Nuqta garmonik tebranmoqda. Nuqtaning eng katta siljishi $x_{\max}=10\text{sm}$, eng katta tezligi $v_{\max}=20\text{ sm/s}$ ga teng. Tebranishning doimo

takrorlanib turuvchi sikl chastotasi ω va maksimal tezlanishi topilsin. ($\omega=2s^{-1}$; $a=40sm/s^2$).

163. Ikkita kamerton bir vaqtda ovoz chiqarmoqda. Ularning tebranish chastotalari ν_1 va ν_2 mos ravishda 440 va 440,5 Hnga teng. Tepkili tebranish davri T aniqlansin. ($T=2s$).

164. Moddiy nuqta bir vaqtning o'zida $x=A_1\cos\omega t$ va $u=A_2\cos 2\omega t$ tenglamalar bilan ifodalanuvchi ikkita o'zaro tik tebranishlarda ishtirok etadi, bunda $A_1=2sm$, $A_2=1sm$. Traektoriya tenglamasi topilsin. ($u=-2(A_2/A_1)\cdot x^2+A_2$; $y=-\frac{1}{2}x^2+1$).

165. Prujinaga osilgan $m=250g$ massali yuk $T=1s$ davr bilan tik yo'nalishda tebranadi. Prujinaning bikrligi K aniqlansin. ($K=9,87N/m$).

166. Agar qurilmaning xususiy tebranishlar davri $T=1s$ va tebranishning logariflik dekrementi $\lambda=0,628$ bo'lsa, so'navchi tebranishlarning davri T topilsin. ($T=1,005s$).

167. Tebranish tizimi $\nu=1000$ Hn chastotali so'navchi tebranishlarni bajarmoqda. Agar rezonans chastota $\nu_{rez}=998$ Hn bo'lsa, xususiy tebranishlar chastotasi ν_0 aniqlansin. ($\nu_0=1002Hz$).

168. $\nu_1=400Hn$ va $\nu_2=600Hn$ chastotalarda majburiy garmonik tebranishlar amplitudasi bir-biriga teng. Rezonans chastota ν_{rez} aniqlansin. So'nish hisobga olinmasin. ($\nu_{rez}=510Hz$).

169. $R=30sm$ radiusli bir jinsli disk uning silindrik sirtining tashkil etuvchilaridan biri orqali o'tuvchi gorizontal o'q atrofida tebranmoqda. Uning tebranish davri T qanday. $\left(T = 2\pi \sqrt{\frac{3R}{2g}} = 1,35s \right)$.